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RADIAL PRESSURE ON A GUN LAUNCHED MOTOR CASE DUE TO SLUMPING PROPELLANT

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An investigation was undertaken to determine the radial pressure due to the slumping propellant in a gun-launched rocket motor case subject to high axial g loading during launch. Using a Rayleigh-Ritz formulation an approximate solution was obtained.

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NOTATION

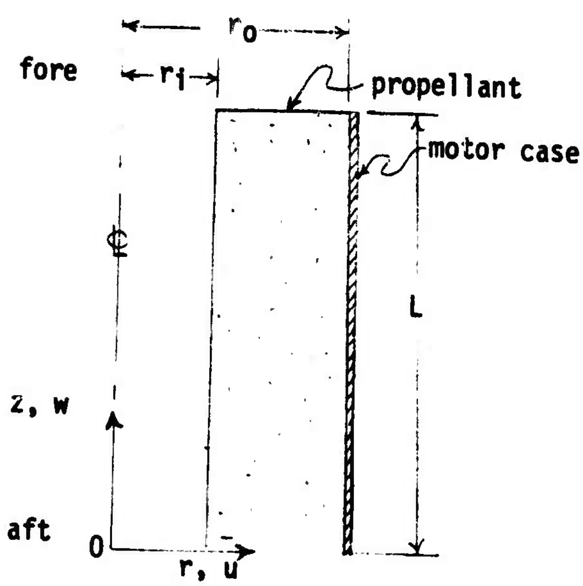
a_m	maximum rigid body acceleration of the projectile
A, B	material constants, eq. (8)
C_i	constants due to body forces
e	dilatation
$e_{rr}, e_{zz}, e_{\theta\theta}$	normal strain components
E	Young's modulus of elasticity
f_z	body force in the z direction
F_{ij}	coefficients of eqs. (21) from Rayleigh-Ritz method
G	shear modulus
L	length of propellant
r, r_i, r_o	radius, inner radius, outer radius
T	total potential energy
u	displacement along r axis
U	elastic strain energy
V	potential energy of external forces
w	displacement along z axis
$\alpha,\beta,\gamma,\delta,\eta,\lambda,\xi,\zeta$	Rayleigh Ritz undetermined coefficients
γ_{rz}	shear strain component
ρ	weight per unit volume
$\sigma_{zz},\sigma_{rr},\sigma_{\theta\theta}$	normal stress components
τ_{rz}	shear stress component

INTRODUCTION

The launch phase of a gun-launched rocket is associated with high acceleration loading of up to 10,000 g. As a result of the high acceleration the motor case portion of the rocket, which contains the propellant, is subjected to axial and lateral loads. The lateral loads are associated with the deformation of the contained propellant. This "slumping propellant" behavior is the concern of the present investigation.

ANALYSIS

The propellant-motor case system may be idealized as a two-dimensional system because of axial symmetry. With cylindrical coordinates, r and z, a typical section is shown below.



u = displacement in the r direction

w = displacement in the z direction

r_i = inside radius of propellant

r_o = outside radius of propellant

L = length of propellant

The particular problem of the unbonded propellant is considered here. The stiff motor case shell is assumed to completely restrain axial displacement at the aft end (r,0) as well as radial displacement along the outer radius (r_o,z) with the resultant boundary conditions,

$$w(r, 0) = 0 \qquad \tau_{rz}(r,0) = 0 \tag{1a}$$

$$u(r_o,z) = 0 \qquad \tau_{rz}(r_o,z) = 0 \tag{1b}$$

The remaining sides (r, L) and (r_i,z) are stress free, that is

$$\sigma_z(r,L) = 0 \qquad \tau_{rz}(r,L) = 0 \tag{1c}$$

$$\sigma_r(r_i,z) = 0 \qquad \tau_{rz}(r_i,z) = 0 \tag{1d}$$

The acceleration of the propellant subjects every particle to the body force

$$f_z = -\rho a_m \tag{2}$$

where ρ is the weight/unit volume, and a_m is the maximum acceleration in g's.

A solution to the problem is obtained by the Rayleigh-Ritz method. According to this method, an approximate solution satisfying the displacement boundary conditions is formed. The unknown coefficients of the solution are determined by minimizing the total potential energy of the system.

Here the displacement fields

$$u(r,z) = (L-z)(\alpha + \beta r + \gamma r^2 + \delta r^3) = (L-z)f(r) \tag{3}$$
$$w(r,z) = z(L - \tfrac{z}{2})(\eta + \lambda r + \xi r^2 + \zeta r^3) = z(L - \tfrac{z}{2})g(r)$$

are assumed. The linear z term in the u displacement and the quadratic z term in the w displacement were chosen to satisfy the following conditions:

i) Results from the Rohm and Haas finite element solution, ref. 1, Appendix B, for the accelerated, unbonded propellant show σ_r and σ_z to be linear functions of z. Note that Equations (3) give this condition, i.e.

$$\sigma_r = \frac{E}{1-\nu^2}\left[\frac{\partial u}{\partial r} + \nu\left(\frac{u}{r} + \frac{\partial w}{\partial z}\right)\right] = \frac{E}{1-\nu^2}\left[\frac{\partial f}{\partial r} + \nu\left\{\frac{f(r)}{r} + g(r)\right\}\right](L-z)$$

$$\sigma_z = \frac{E}{1-\nu^2}\left[\frac{\partial w}{\partial z} + \nu\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right)\right] = \frac{E}{1-\nu^2}\left[g + \nu\left\{\frac{\partial f}{\partial r} + \frac{f(r)}{r}\right\}\right](L-z)$$

ii) The propellant is assumed to be nearly incompressible, and thus linearly varying compression along the z coordinate causes the propellant to move towards the centerline since it is restrained by the case from moving outward, i.e. $u \leq 0$ for all z, and $u = 0$ for $z = L$. This assumed displacement field is not valid for the bonded propellant. The equation for w satisfies the displacement boundary condition $w(r,0) = 0$. The displacement boundary condition, $u(r_0,z) = 0$ is satisfied by setting

$$\alpha = -\beta r_0 - \gamma r_0^2 - \delta r_0^3 \tag{4}$$

The total potential energy of the system, T, resulting from the displacement fields given by equations (3) must be formed. Denoting the elastic strain energy as U and the potential energy of external forces as V, we have

$$T = U + V \tag{5}$$

Determination of the elastic strain energy U proceeds as follows. According to the strain-displacement relations

$$e_r = \frac{\partial u}{\partial r} = (L - z)(\beta + 2\gamma r + 3\delta r^2)$$

$$e_\theta = \frac{u}{r} = (L - z)(\frac{\alpha}{r} + \beta + \gamma r + \delta r^2)$$

$$e_z = \frac{\partial w}{\partial z} = (L - z)(\eta + \lambda r + \xi r^2 + \zeta r^3) \tag{6}$$

$$\gamma_{rz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} = -(\alpha + \beta r + \gamma r^2 + \delta r^3)$$

$$+z(\frac{L}{2} - z)(\lambda + 2\xi r + 3\zeta r^2)$$

and the stress-strain relations,

$$\sigma_r = Ae_r + B(e_z + e_\theta)$$

$$\sigma_r = Ae_\theta + B(e_r + e_z) \tag{7}$$

$$\sigma_z = Ae_z + B(e_r + e_\theta)$$

$$\tau_{rz} = G\gamma_{rz}$$

where

$$A = \frac{(1 - \nu)E}{(1 + \nu)(1 - 2\nu)}$$

$$B = \frac{\nu E}{(1 + \nu)(1 - 2\nu)} \tag{8}$$

and

$$G = \frac{E}{2(1 + \nu)} \tag{9}$$

Substituting equations (6) into (7) yields stress-displacement equations. The elastic strain energy of the system is given by

$$U = \frac{1}{2} \iiint \sigma_{ij} e_{ij} \, dV = \frac{1}{2} \iiint (\sigma_r e_r + \sigma_\theta e_\theta + \sigma_z e_z + \tau_{rz}\gamma_{rz}) \, dV \tag{10}$$

where the integration is over the volume of the propellant. The independence with the θ coordinate gives

$$U = 2\pi \cdot \frac{1}{2} \int_{r_o}^{r_i} \int_{o}^{L} (\sigma_r e_r + \sigma_\theta e_\theta + \sigma_z e_z + \tau_{rz}\gamma_{rz}) \, r \, dr \, dz \tag{11}$$

For convenience, let

$$\begin{aligned} U_a &= \pi \iint \sigma_r e_r \, r \, dr \, dz \\ U_b &= \pi \iint \sigma_\theta e_\theta \, r \, dr \, dz \\ U_c &= \pi \iint \sigma_z e_z \, r \, dr \, dz \\ U_d &= \pi \iint \tau_{rz}\gamma_{rz} \, r \, dr \, dz \end{aligned} \tag{12}$$

then

$$U = U_a + U_b + U_c + U_d \tag{13}$$

Substituting the stress-displacement, and strain displacement relations into equation (12), and integrating, we obtain

$$U_a = \frac{\pi L^3}{3}\left[A\left\{\frac{1}{2}\beta^2 r^2 + \frac{1}{2}\gamma^2 r^4 + \frac{3}{2}\delta^2 r^6 + \frac{4}{3}\beta\gamma r^3 + \frac{9}{4}\beta\delta r^4 + \frac{12}{5}\gamma\delta r^5\right\} + B\left\{\alpha\beta r + \frac{r^2}{2}(\beta\eta + \beta^2 + 2\gamma\alpha) + \frac{r^3}{3}(3\beta\gamma + 2\gamma\eta + 3\delta\alpha + \beta\lambda) + \frac{r^4}{4}(4\beta\delta + 2\gamma^2 + 3\delta\eta + 2\gamma\lambda + \beta\xi) + \frac{r^5}{5}(5\gamma\delta + 3\delta\lambda + \beta\zeta + 2\gamma\xi) + \frac{r^6}{6}(3\delta^2 + 3\delta\xi + 2\gamma\zeta) + \frac{3}{7}\delta\zeta r^7\right\}\right]_{r_i}^{r_o} \tag{14}$$

$$U_b = \frac{\pi L^3}{3}\left[A\left\{\alpha^2 \ln r + 2\alpha\beta r + \frac{r^2}{2}(\beta^2 + 2\alpha\gamma) + \frac{r^3}{3}(2\alpha\delta + 2\beta\gamma) + \frac{r^4}{4}(\gamma^2 + 2\beta\delta) + \frac{2}{5}\gamma\delta r^5 + \frac{1}{6}\delta^2 r^6\right\} + B\left\{r(\alpha\eta + \alpha\beta) + \frac{r^2}{2}(2\alpha\gamma + \beta\eta + \beta^2 + \alpha\lambda) + \frac{r^3}{3}(3\alpha\delta + 3\beta\gamma + \gamma\eta + \beta\lambda + \alpha\xi) + \frac{r^4}{4}(4\beta\delta + 2\gamma^2 + \delta\eta + \gamma\lambda + \beta\xi + \alpha\zeta) + \frac{r^5}{5}(5\gamma\delta + \delta\lambda + \gamma\xi + \beta\zeta) + \frac{r^6}{6}(3\delta^2 + \delta\xi + \gamma\zeta) + \delta\zeta\frac{r^7}{7}\right\}\right]_{r_i}^{r_o} \tag{15}$$

$$U_c = \frac{\pi L^3}{3}\left[A\left\{\frac{r^2}{2}\eta^2 + \frac{2}{3}\eta\lambda r^3 + \frac{r^4}{4}(\lambda^2 + 2\eta\xi) + \frac{r^5}{5}(2\lambda\xi + 2\eta\zeta) + \frac{r^6}{6}(\xi^2 + 2\lambda\zeta) + \frac{3}{7}\xi\zeta r^7 + \zeta^2\frac{r^8}{8}\right\} + B\left\{\alpha\eta r + \frac{r^2}{2}(2\beta\eta + \alpha\lambda) + \frac{r^3}{3}(3\gamma\eta + 2\beta\lambda + \alpha\xi) + \frac{r^4}{4}(4\delta\eta + 3\gamma\lambda + 2\beta\xi + \alpha\zeta) + \frac{r^5}{5}(4\delta\lambda + 3\gamma\xi + 2\beta\zeta) + \frac{r^6}{6}(4\delta\xi + 3\gamma\zeta) + \frac{4}{7}\delta\zeta r^7\right\}\right]_{r_i}^{r_o} \tag{16}$$

$$
\begin{aligned}
U_d = \pi G \Big[& L\Big\{ \tfrac{1}{2}\alpha^2 r^2 + \tfrac{2}{3}\alpha\beta r^3 + \tfrac{r^4}{4}(\beta^2 + 2\alpha\gamma) + \tfrac{r^5}{5}(2\alpha\delta \\
& \quad + 2\gamma\beta) + \tfrac{1}{6}\beta\delta r^6 + \tfrac{1}{7}\gamma\delta r^7 \Big\} \\
& - \tfrac{L^3}{12}\Big\{ \tfrac{1}{2}\alpha\lambda r^2 + \tfrac{r^3}{3}(2\alpha\xi + \beta\lambda) + \tfrac{r^4}{4}(3\alpha\zeta + 2\beta\xi \\
& \quad + \gamma\lambda) + \tfrac{r^5}{5}(3\beta\zeta + 2\gamma\xi + \delta\lambda) + \tfrac{r^6}{6}(3\gamma\zeta \\
& \quad + 2\xi\delta) + \tfrac{3}{7}\delta\zeta r^7 \Big\} \\
& + \tfrac{L^5}{30}\Big\{ \tfrac{1}{2}\lambda^2 r^2 + \tfrac{4}{3}\lambda\xi r^3 + r^4(\xi^2 + \tfrac{3}{2}\lambda\zeta) + \tfrac{12}{5}\xi\zeta r^5 \\
& \quad + \tfrac{3}{2}\zeta^2 r^6 \Big\} \Big]_{r_i}^{r_o}
\end{aligned}
\qquad (17)
$$

The potential energy of external forces V, associated with the inertia body force f_z given by equation (2) is,

$$
V = -\iiint f_z \cdot w \, dV = -2\pi \int_{r_i}^{r_o} \int_{0}^{L} \rho \, a_m \cdot w \cdot r \, dr \, dz \qquad (18)
$$

Substituting the expression for w from equation (3) into equation (18) yields

$$
V = -\frac{\pi L^3}{3} \rho \, a_m \cdot \left[\eta r^2 + \frac{2}{3}\lambda r^3 + \frac{1}{2}\xi r^4 + \frac{2}{5}\zeta r^5\right]_{r_i}^{r_o} \qquad (19)
$$

The total potential energy of the system is obtained as

$$
T = U_a + U_b + U_c + U_d + V \qquad (20)
$$

The theo.em of minimum potential energy states that equilibrium is associated with a minimum of T. Here T is a function of the independent parameters $\beta, \gamma, \delta, \eta, \xi$ and ζ. The minimum of T is obtained by taking the partial derivative of T with respect to each of these seven coefficients. This yields the system of 7 linear algebraic equations in 7 unknowns,

$$\begin{bmatrix} F_{11} & F_{12} & \cdots\cdots & F_{17} \\ F_{21} & F_{22} & \cdots\cdots & F_{27} \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ F_{71} & F_{72} & & F_{77} \end{bmatrix} \begin{Bmatrix} \beta \\ \gamma \\ \delta \\ \eta \\ \lambda \\ \xi \\ \zeta \end{Bmatrix} = \begin{Bmatrix} C_1 \\ C_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ C_7 \end{Bmatrix} \tag{21}$$

The explicit expressions for the F_{ij} and C_i (i_{ij} = 1, 7) coefficients are

$$F(1,1) = A\left\{-2r_0^2 - 2r_i^2 + 4r_0 r_i + 2r_0^2 \ln\frac{r_0}{r_i}\right\} + B\left\{-2r_0^2 - 2r_i^2\right\} + 3G\left\{\frac{1}{6}r_0^4 - r_0^2 r_i^2 + \frac{4}{3}r_0 r_i^3 - \frac{1}{2}r_i^4\right\} L^2$$

$$F(1,2) = A\left\{-r_0^3 - 2r_i^3 + 2r_0^2 r_i + r_0 r_i^2 + 2r_0^3 \ln\frac{r_0}{r_i}\right\} + B\left\{-2r_0^3 + 2r_0 r_i^2 + 2r_0^2 r_i - 2r_i^3\right\} + 3G/L^2\left\{\frac{7}{30}r_0^5 - r_0^3 r_i^2 + \frac{2}{3}r_0^2 r_i^3 + \frac{1}{2}r_0 r_i^4 - \frac{2}{5}r_i^5\right\}$$

$$F(2,1) = F(1,2)$$

$$F(1,3) = F(3,1) = A\left\{\tfrac{1}{12} r_0^4 - \tfrac{11}{4} r_i^4 + 2 r_0^3 r_i + \tfrac{2}{3} r_0 r_i^3 + 2 r_0^4 \ln \frac{r_0}{r_i}\right\}$$
$$+ B\left\{-2 r_0^4 + 2 r_0 r_i^3 - 2 r_i^4 + 2 r_0^3 r_i\right\}$$
$$+ 3G/L^2 \left\{\tfrac{4}{15} r_0^6 - r_0^4 r_i^2 + \tfrac{2}{3} r_0^3 r_i^3 + \tfrac{2}{5} r_0 r_i^5 - \tfrac{1}{3} r_i^6\right\}$$

$$F(1,4) = F(4,1) = B\left\{-2 r_i^2 + 2 r_0 r_i\right\}$$

$$F(1,5) = F(5,1) = B\left\{\tfrac{4}{3}(r_0^3 - r_i^3) - r_0^3 + r_0 r_i^2\right\}$$
$$-2G\left\{-\tfrac{1}{6} r_0^3 + \tfrac{1}{2} r_0 r_i^2 - \tfrac{1}{3} r_i^3\right\}$$

$$F(1,6) = F(6,1) = B\left\{\tfrac{1}{3} r_0^4 + \tfrac{2}{3} r_0 r_i^3 - r_i^4\right\}$$
$$-2G\left\{-\tfrac{1}{6} r_0^4 + \tfrac{2}{3} r_0 r_i^3 - \tfrac{1}{2} r_i^4\right\}$$

$$F(1,7) = F(7,1) = B\left\{\tfrac{3}{10} r_0^5 + \tfrac{1}{2} r_0 r_i^4 - \tfrac{4}{5} r_i^5\right\}$$
$$-2G\left\{-\tfrac{3}{20} r_0^5 + \tfrac{3}{4} r_0 r_i^4 - \tfrac{3}{5} r_i^5\right\}$$

$$F(2,2) = A\left\{-\tfrac{1}{2} r_0^4 - \tfrac{3}{2} r_i^4 + 2 r_0^2 r_i^2 + 2 r_0^4 \ln \frac{r_0}{r_i}\right\}$$
$$+ B\left\{-2 r_0^4 + 4 r_0^2 r_i^2 - 2 r_i^4\right\}$$
$$+ 3G/L^2 \left\{\tfrac{1}{3} r_0^6 - r_0^4 r_i^2 + r_0^2 r_i^4 - \tfrac{1}{3} r_i^6\right\}$$

$$F(2,3) = F(3,2) = A\left\{\tfrac{17}{15} r_0^5 - \tfrac{14}{5} r_i^5 + \tfrac{2}{3} r_0^2 r_i^3 + r_0^3 r_i^2 + 2 r_0^5 \ln \frac{r_0}{r_i}\right\}$$
$$+ B\left\{-2 r_0^5 + 2 r_0^3 r_i^2 + 2 r_0^2 r_i^3 - 2 r_i^5\right\}$$
$$+ \frac{3G}{L^2}\left\{\tfrac{27}{70} r_0^7 - r_0^5 r_i^2 + \tfrac{1}{2} r_0^3 r_i^4 + \tfrac{2}{5} r_0^2 r_i^5 - \tfrac{2}{7} r_i^7\right\}$$

$$F(2,4) = F(4,2) = B\left\{-2 r_i^3 + 2 r_0^2 r_i\right\}$$

$$F(2,5) = F(5,2) = B\left\{\tfrac{3}{2}(r_0^4 - r_i^4) - r_0^4 + r_0^2 r_i^2\right\}$$
$$-2G\left\{-\tfrac{1}{4} r_0^4 + \tfrac{1}{2} r_0^2 r_i^2 - \tfrac{1}{4} r_i^4\right\}$$

$$F(2,6) = F(6,2) = B\left\{\frac{8}{15} r_o^5 + \frac{2}{3} r_o^2 r_i^3 - \frac{6}{5} r_i^5\right\}$$
$$-2G\left\{-\frac{4}{15} r_o^5 + \frac{2}{3} r_o^2 r_i^3 - \frac{2}{5} r_i^5\right\}$$

$$F(2,7) = F(7,2) = B\left\{\frac{1}{2} r_o^6 + \frac{1}{2} r_o^2 r_i^4 - r_i^6\right\}$$
$$-2G\left\{-\frac{1}{4} r_o^6 + \frac{3}{4} r_o^2 r_i^4 - \frac{1}{12} r_i^6\right\}$$

$$F(3,3) = A\left\{2 r_o^6 - \frac{10}{3} r_i^6 + \frac{4}{3} r_o^3 r_i^3 + 2 r_o^6 \ln \frac{r_o}{r_i}\right\}$$
$$+B\left\{-2 r_o^6 - 2 r_i^6 + 4 r_o^3 r_i^3\right\}$$
$$+\frac{3G}{L^2}\left\{\frac{1}{20} r_o^8 - r_o^6 r_i^2 + \frac{4}{5} r_o^3 r_i^5 - \frac{1}{4} r_i^8\right\}$$

$$F(3,4) = F(4,3) = B\left\{-2 r_i^4 + 2 r_o^3 r_i\right\}$$

$$F(3,5) = F(5,3) = B\left\{\frac{8}{5}\left(r_o^5 - r_i^5\right) - r_o^5 + r_o^3 r_i^2\right\}$$
$$-2G\left\{-\frac{3}{10} r_o^5 + \frac{1}{2} r_o^3 r_i^2 - \frac{1}{5} r_i^5\right\}$$

$$F(3,6) = F(6,3) = B\left\{\frac{2}{3} r_o^6 + \frac{2}{3} r_o^3 r_i^3 - \frac{4}{3} r_i^6\right\}$$
$$-2G\left\{-\frac{1}{3} r_o^6 + \frac{2}{3} r_o^3 r_i^3 - \frac{1}{3} r_i^6\right\}$$

$$F(3,7) = F(7,3) = B\left\{\frac{9}{14} r_o^7 + \frac{1}{2} r_o^3 r_i^4 - \frac{8}{7} r_i^7\right\}$$
$$-2G\left\{-\frac{9}{28} r_o^7 + \frac{3}{4} r_o^3 r_i^4 - \frac{3}{7} r_i^7\right\}$$

$$F(4,4) = A\left\{r_o^2 - r_i^2\right\}$$

$$F(4,5) = F(5,4) = A\cdot\left\{\frac{2}{3}\left(r_o^3 - r_i^3\right)\right\}$$

$$F(4,6) = F(6,4) = A\cdot\left\{\frac{1}{2}\left(r_o^4 - r_i^4\right)\right\}$$

$$F(4,7) = F(7,4) = A\left\{\tfrac{1}{2}(\Lambda_o^4 - \Lambda_i^4)\right\}$$

$$F(5,5) = A\left\{\tfrac{1}{2}(\Lambda_o^4 - \Lambda_i^4)\right\} + GL^2\left\{\tfrac{2}{5}(\Lambda_o^2 - \Lambda_i^2)\right\}$$

$$F(5,6) = F(6,5) = A\left\{\tfrac{2}{5}(\Lambda_o^5 - \Lambda_i^5)\right\} + GL^2\left\{\tfrac{8}{15}(\Lambda_o^3 - \Lambda_i^3)\right\}$$

$$F(5,7) = F(7,5) = A\left\{\tfrac{1}{3}(\Lambda_o^6 - \Lambda_i^6)\right\} + GL^2\left\{\tfrac{3}{5}(\Lambda_o^4 - \Lambda_i^4)\right\}$$

$$F(6,6) = A\left\{\tfrac{1}{3}(\Lambda_o^6 - \Lambda_i^6)\right\} + GL^2\left\{\tfrac{4}{5}(\Lambda_o^4 - \Lambda_i^4)\right\}$$

$$F(6,7) = F(7,6) = A\left\{\tfrac{2}{7}(\Lambda_o^7 - \Lambda_i^7)\right\} + GL^2\left\{\tfrac{24}{25}(\Lambda_o^5 - \Lambda_i^5)\right\}$$

$$F(7,7) = A\left\{\tfrac{1}{4}(\Lambda_o^8 - \Lambda_i^8)\right\} + GL^2\left\{\tfrac{6}{5}(\Lambda_o^6 - \Lambda_i^6)\right\}$$

$$C_1 = C_2 = C_3 = 0$$

$$C_4 = -\rho a_m(\Lambda_o^2 - \Lambda_i^2) \qquad C_5 = -2\rho a_m(\Lambda_o^3 - \Lambda_i^3)/3.$$

$$C_6 = -\rho a_m(\Lambda_o^4 - \Lambda_i^4)/2. \qquad C_7 = -2\rho a_m(\Lambda_o^5 - \Lambda_i^5)/5.$$

A computer program was used to obtain the solution of the problem. A large number of problems were solved for various values of the physical parameters, r_i, r_o, L and a_m, and the propellant properties ν and E. A listing of the program is given in Appendix A.

DISCUSSION OF RESULTS

It has been noted by other investigators that numerical difficulties arise in the analysis of propellants when Poisson's ratio approaches 0.5

(ref. 2). To circumvent this difficulty Hermann (ref. 3) gives a special variational theorem for nearly incompressible materials. In the present investigation the numerical problem was overcome by using a CDC 6600 computer with double precision. This results in approximately 32 significant digits, sufficient for Poisson's ratio to 0.4999999.

A study was conducted on the effect of Poisson's ratio on the propellant stress for a propellant with 1.5 inch outer radius, 0.5 inch inner radius, 9.5 inch length, 500,000 psi Young's modulus and a_m equal to 8000 g. Poisson's ratio was varied between 0.45 and 0.4999999. The computer program results for σ_r at the propellant-Case interface at the base are given in Table 1.

Table 1

ν	σ_r(psi)	$\Delta\nu$	$\Delta\sigma$	e
.45	-1589			-2.0×10^{-3}
.48	-1691	.03	-112	-8.9×10^{-4}
.49	-1771	.01	- 80	-2.1×10^{-4}
.499	-2513	.004	-616	-2.8×10^{-5}
.4999	-3744	.0009	-1231	-4.0×10^{-6}
.49999	-4469	.0009	-725	-4.8×10^{-7}
.499999	-4613	.00009	-144	-5.0×10^{-8}
.4999999	-4630	.000009	- 27	-5.0×10^{-9}

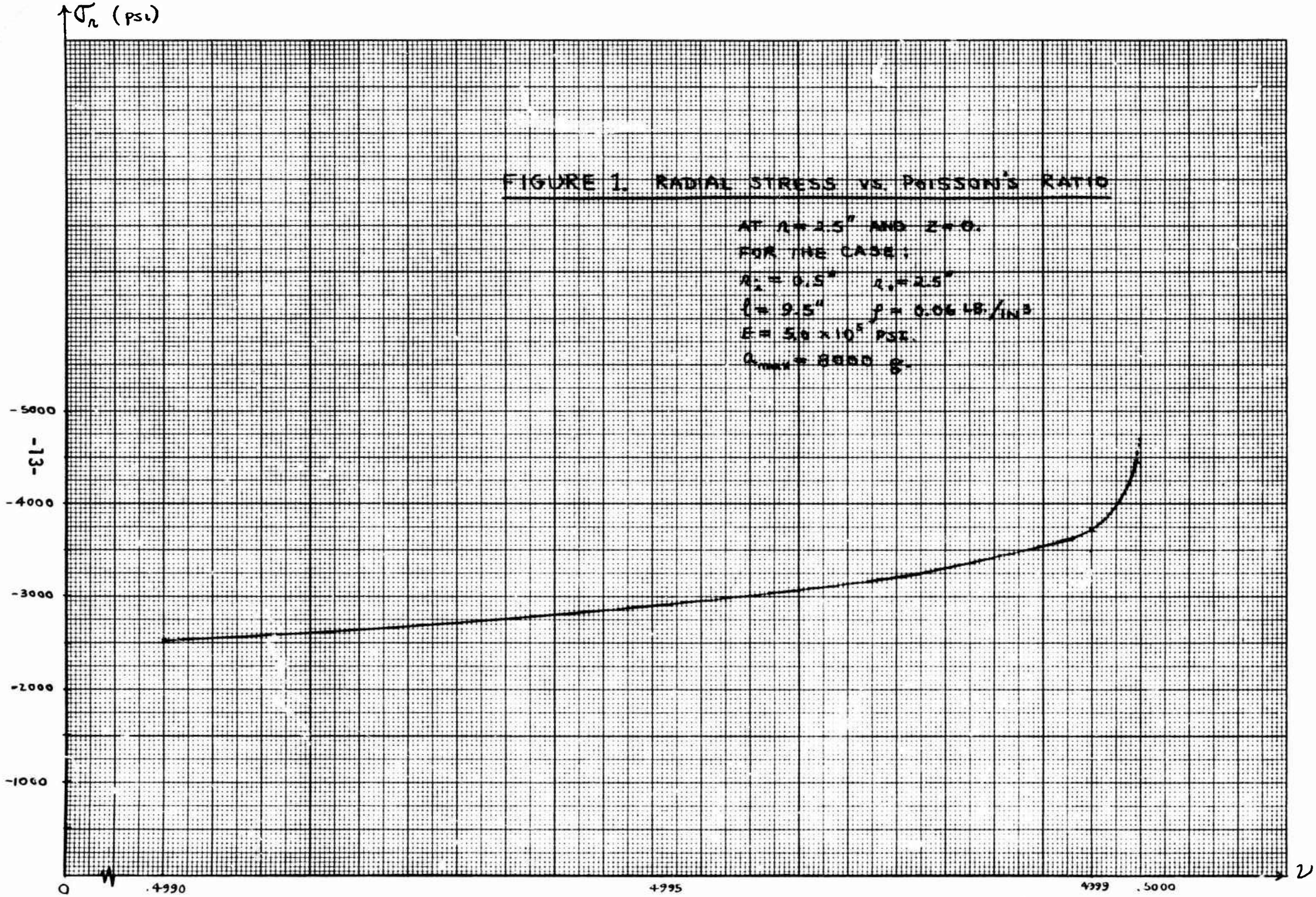


FIGURE 1. RADIAL STRESS VS. POISSON'S RATIO

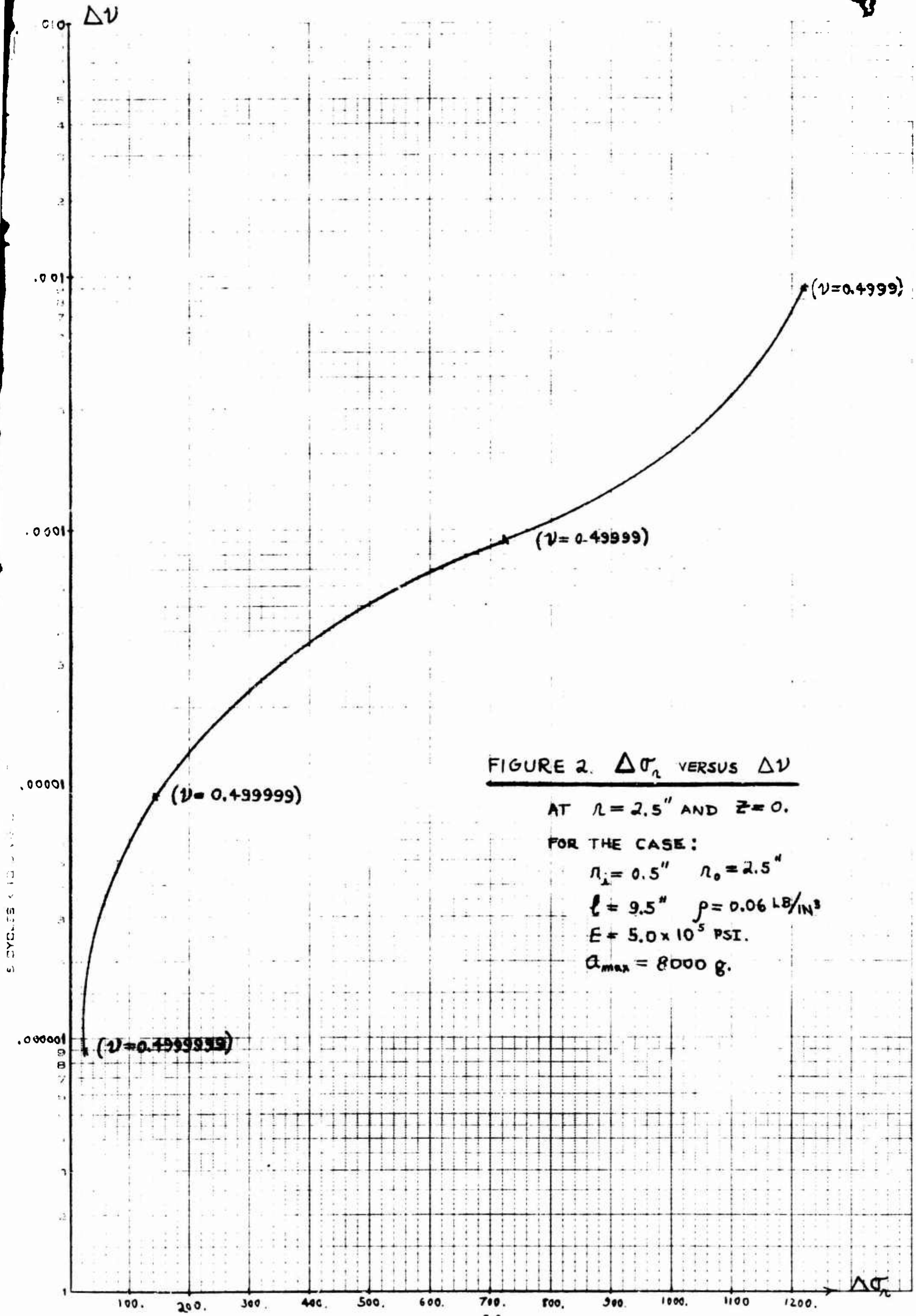


FIGURE 2. $\Delta\sigma_r$ VERSUS $\Delta\nu$

AT $r = 2.5''$ AND $z = 0$.

FOR THE CASE:

$r_i = 0.5''$ $r_o = 2.5''$

$\ell = 9.5''$ $\rho = 0.06$ LB/IN^3

$E = 5.0 \times 10^5$ PSI.

$a_{max} = 8000$ g.

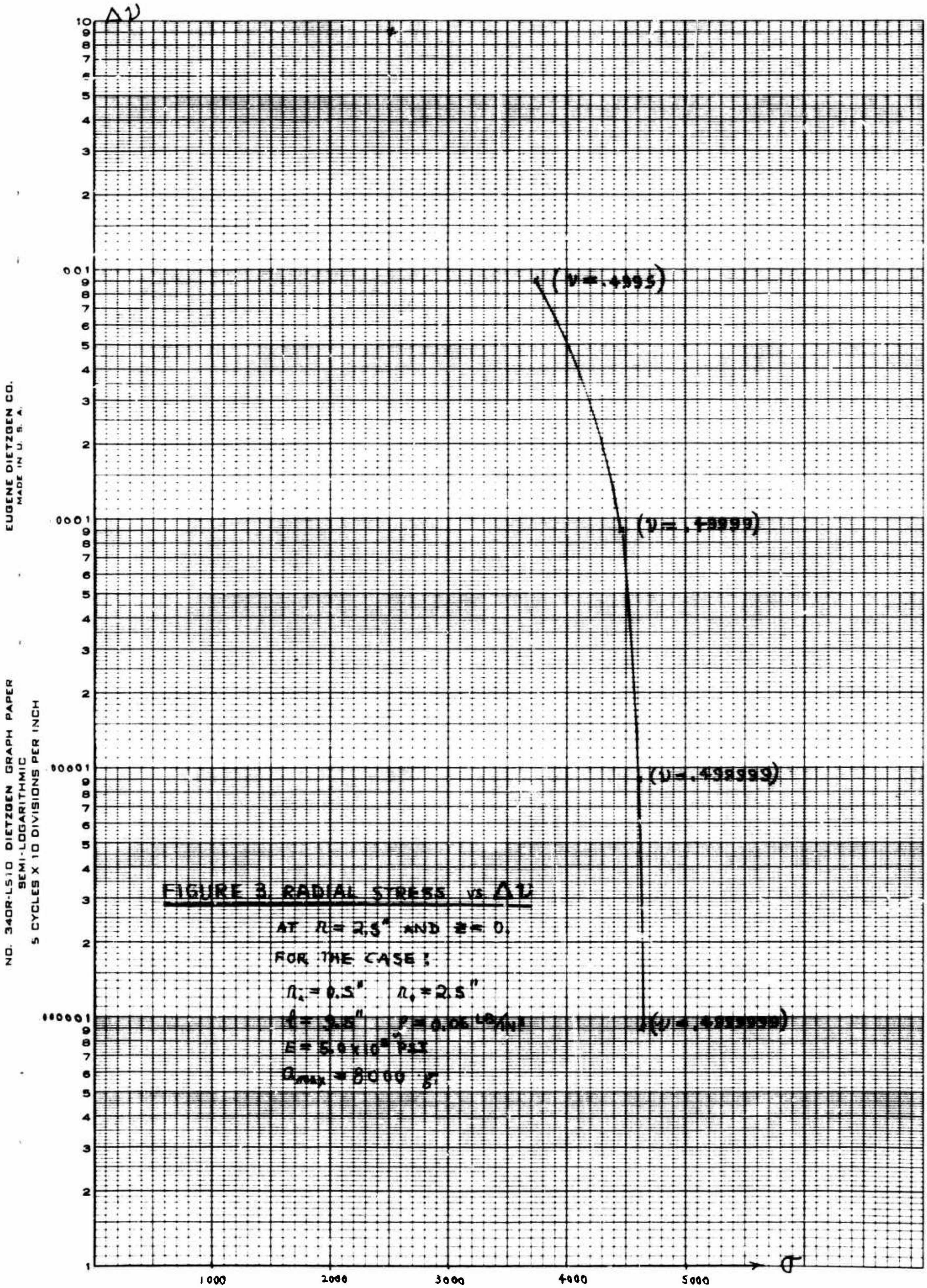


FIGURE 3. RADIAL STRESS vs Δν

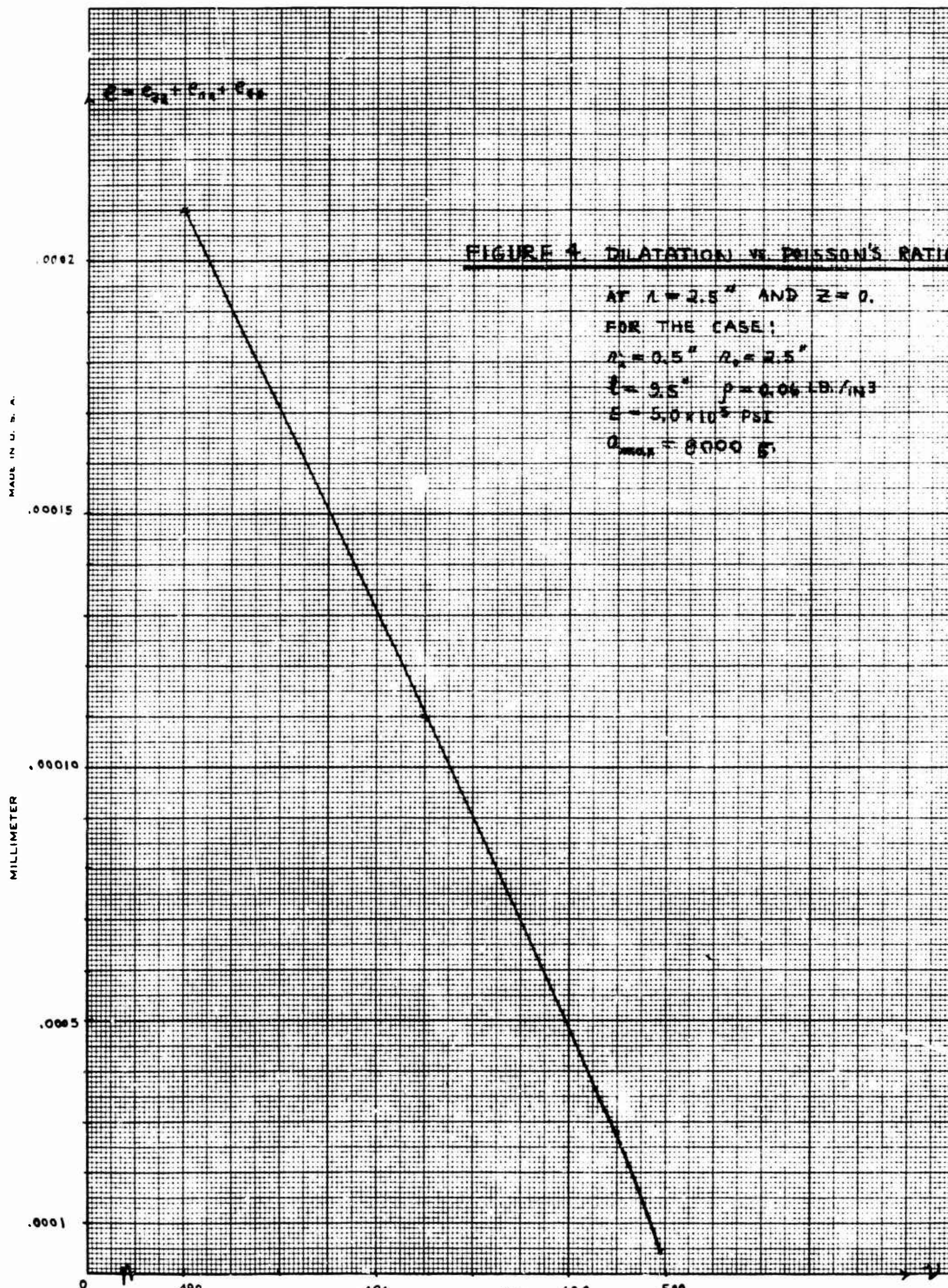


FIGURE 4. DILATATION vs POISSON'S RATIO

AT $r = 2.5''$ AND $z = 0$.

FOR THE CASE:

$r_i = 0.5''$ $r_o = 2.5''$

$\ell = 9.5''$ $\rho = 0.06$ LB./IN^3

$E = 5.0 \times 10^3$ PSI

$a_{max} = 8000$ g

Figures 1 through 3 are curves showing these results; the first figure being a direct plot of stress σ_r versus Poisson's ratio ν. Figures 2 and 3 show more clearly that the stress approaches a finite value of -4650 psi as Poisson's ratio approaches 0.5. In Figure 4 the dilatation $e = e_\theta + e_r + e_z$, at $r = r_0$ and $z = 0$ is plotted against Poisson's ratio. The resulting curve verifies that the incompressibility condition e approaches zero as ν approaches 1/2 is satisfied.

In addition to the study on the effect of Poisson's ratio on propellant behavior, analyses were made to determine the effect of other parameters to behavior. The results obtained show that the radial stress varies linearly with respect to acceleration, radii, length and Young's modulus of elasticity.

The results of this analysis were compared with two other sources; a Rohm and Haas finite element analysis (ref. 3), and an exact solution for an incompressible material with zero interior radius. The present formulation resulted in a 20 percent higher radial stress than the finite element program, and 2 percent higher than the exact solution, at the propellant-case interface at the base.

CONCLUSIONS

A computer program was developed for the determination of the stress distribution due to a slumping propellant contained within a launched rigid motor case. The program was initially incorporated into the SATANS finite difference program for the calculation of the elastic static buckling load of the motor case subjected to high acceleration loading. The stiffening of the motor case, which is predicted by theory, was obtained in the SATANS analysis; a favorable structural response.

An unfavorable effect on structural integrity is associated with the additional contribution to yield from the propellant radial stress. For a motor case of 2.5 inch outer radius, zero inner radius, 1/8 inch thickness, 20 inch length, and a maximum acceleration of 8,000 g, the radial stress of approximately 10,000 psi gives a hoop stress of 100,000 psi. The contribution to yield of this stress component is of significant magnitude. That is, the propellant has a beneficial buckling stiffening effect but an adverse affect on yield.

The present analysis is based on the linear elastic theory of solids and, therefore, the results are not valid beyond the initial yield point of the propellant. Because it is difficult to obtain the material properties of propellants (Young's modulus, Poisson's ratio, and initial yield), definite assertions regarding the structural integrity of the propellant are not given here; however, the stress results obtained in some of the analyses show that additional study regarding this concern may be warranted.

REFERENCES

1. Ball, Robert E., and Salinas, David, "Analysis of a Three Inch Gun Launched Finned Motor Case," Naval Postgraduate School, NPS-57Bp-72011A, January 1972.

2. Thacker, J. H., "Deformation of Case-Bonded Propellants Under Axial Acceleration," 20th Meeting Bulletin, JANAF-ARPA-NASA Panel on Physical Properties of Solid Propellants, I, Nov. 1961, 81-90.

3. Hermann, L. R., "Elasticity Equations for Incompressible and Nearly Incompressible Materials by a Variational Theorem," AIAA Journal, Vol. 3, No. 11, Oct. 1965, 1896-1900.

```
C
C     THIS PROGRAM CALCULATES THE STRESSES IN A SLUMPING PROPELLANT
C     DUE TO ACCELERATION LOADING
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION F(7,7),S(7),Y(7),T(7,7),FT(7,7),SIGXX(15),R(15)
C
      READ(5,5) NLOOP,NR
    5 FORMAT(2I10)
      WRITE(6,6) NLOOP
      READ(5,8) (R(K),K=1,NR)
    8 FORMAT(8F10.5)
      WRITE(6,9) (R(K),K=1,NR)
    9 FORMAT(5X,8F10.5)
    6 FORMAT(1H05X,'NUMBER OF PROBLEMS IS',I10)
      DO 200 NL=1,NLOOP
      WRITE(6,7) NL
    7 FORMAT(1H05X,'PROBLEM ',I10)
C
      READ(5,10) E,ZNU,ZL,AMAX,RO,RI,RHC
   10 FORMAT(7F10.5)
      WRITE(6,15) E,ZNU,ZL,AMAX,RO,RI,RHC
   15 FORMAT(1H05X,'YOUNGS MODULUS=',E17.8, //5X,'POISSONS RATIO=',E17.8
     1, //5X,'PROPELLANT LENGTH=',E17.8, //5X,'MAX. ACCEL.=',E17.8, //5X
     2,'OUTER RADIUS=',E17.8, //5X,'INNER RADIUS=',E17.8, //5X,'SPECIFIC
     3 DENSITY=',E17.8)
C
      A=(1.-ZNU)*E/((1.+ZNU)*(1.-2.*ZNU))
      B=ZNU*E/((1.+ZNU)*(1.-2.*ZNU))
      G=E/(2.*(1.+ZNU))
      WRITE(6,30) A,B,G
   30 FORMAT(2X,'A=',E17.8,10X,'B=',E17.8,10X,'G=',E17.8)
      ZL2=ZL**2
      ROOVRI=RO/RI
C
      RO2=RO**2
      RO3=RO*RO2
      RO4=RO*RO3
      RO5=RO*RO4
      RO6=RO*RO5
      RO7=RO*RO6
      RO8=RO*RO7
C
      RI2=RI**2
      RI3=RI*RI2
      RI4=RI*RI3
      RI5=RI*RI4
```

```
      RI6=RI*RI5
      RI7=RI*RI6
      RI8=RI*RI7
C
      RHS4=-RHO*AMAX*(RO2-RI2)
      RHS5=-2.*RHO*AMAX*(RO3-RI3)/3.
      RHS6=-RHO*AMAX*(RO4-RI4)/2.
      RHS7=-2.*RHO*AMAX*(RO5-RI5)/5.
      WRITE(6,20) RHS4,RHS5,RHS6,RHS7
   20 FORMAT(4E20.8)
C
      F(1,1)=A*(-2.*RO2-2.*RI2+4.*RO*RI+2.*RO2*DLOG(ROOVRI))
     1+B*(-2.*RO2+4.*RO*RI-2.*RI2)+3.*G*(RO4/3.-RO2*RI2+4.*RO*RI3/3.
     2-RI4/2.)/ZL2
      F(1,2)=A*(-RO3-2.*RI3+2.*RO2*RI+RO*RI2+2.*RO3*DLOG(ROOVRI))
     1+B*(-2.*RO3+2.*RO*RI2+2.*RO2*RI-2.*RI3)+3.*G*(7.*RO5/30.-RO3*RI2
     2+2.*RO2*RI3/3.+RO*RI4/2.-2.*RI5/5.)/ZL2
      F(1,3)=A*(RO4/12.-11.*RI4/4.+2.*RO3*RI+2.*RO*RI3/3.+2.*RO4*DLOG(
     1ROOVRI))+B*(-2.*RO4+2.*RO*RI3-2.*RI4+2.*RO3*RI)
     2+3.*G*(4.*RO6/15.-RO4*RI2+2.*RO3*RI3/3.+2.*RO*RI5/5.-RI6/3.)/ZL2
      F(1,4)=B*(-2.*RI2+2.*RO*RI)
      F(2,2)=A*(-RO4/2.-3.*RI4/2.+2.*RO2*RI2+2.*RO4*DLOG(ROOVRI))
     1+B*(-2.*RO4+4.*RO2*RI2-2.*RI4,
     2+3.*G*(RO6/3.-RO4*RI2+RO2*RI4-RI6/3.)/ZL2
      F(2,3)=A*(17.*RO5/15.-14.*RI5/5.+2.*RO2*RI3/3.+RO3*RI2+2.*RO5*DLOG
     1(ROOVRI))+B*(-2.*RO5+2.*RO3*RI2+2.*RO2*RI3-2.*RI5)
     2+3.*G*(27.*RO7/70.-RO5*RI2+RO3*RI4/2.+2.*RO2*RI5/5.-2.*RI7/7.)/ZL2
      F(2,4)=B*(-2.*RI3+2.*RO2*RI)
      F(3,3)=A*(2.*RO6-10.*RI6/3.+4.*RO3*RI3/3.+2.*RO6*DLOG(ROOVRI))
     1+B*(-2.*RO6-2.*RI6+4.*RO3*RI3)
     2+3.*G*(RO8/20.-RO6*RI2+4.*RO3*RI5/5.-RI8/4.)/ZL2
      F(3,4)=B*(-2.*RI4+2.*RO3*RI)
      F(4,4)=A*(RO2-RI2)
      F(1,5)=B*(4.*(RO3-RI3)/3.-RO3+RO*RI2)
      F(1,5)=-2.*G*(-RO3/6.+RO*RI2/2.-RI3/3.)+F(1,5)
      F(1,6)=B*(RO4/3.+2.*RO*RI3/3.-RI4)-2.*G*(-RO4/6.+2.*RO*RI3/3.-
     1RI4/2.)
      F(2,5)=B*(3.*(RO4-RI4)/2.-RO4+RO2*RI2)
      F(2,5)=F(2,5)-2.*G*(-RO4/4.+RO2*RI2/2.-RI4/4.)
      F(2,6)=B*(8.*RO5/15.+2.*RO2*RI3/3.-6.*RI5/5.)-2.*G*(-4.*RO5/15.+2.
     1*RO2*RI3/3.-2.*RI5/5.)
      F(3,5)=B*(8.*(RO5-RI5)/5.-RO5+RO3*RI2)
      F(3,5)=F(3,5)-2.*G*(-3.*RO5/10.+RO3*RI2/2.-RI5/5.)
      F(3,6)=B*(2.*RO6/3.+2.*RO3*RI3/3.-4.*RI6/3.)-2.*G*
     1(-RO6/3.+2.*RO3*RI3/3.-RI6/3.)
      F(4,5)=A*(2.*(RO3-RI3)/3.)
      F(4,6)=A*(RO4-RI4)/2.
      F(5,5)=A*(RO4-RI4)/2.
```

```
      F(5,5)=F(5,5)+2.*G*(ZL**2)*(RO2-RI2)/5.
      F(5,6)=2.*A*(RO5-RI5)/5.+8.*G*(ZL**2)*(RC3-RI3)/15.
      F(6,6)=A*(RO6-RI6)/3.+4.*G*(ZL**2)*(RO4-RI4)/5.
      F(1,7)=B*(3.*RO5/10.+RO*RI4/2.-4.*RI5/5.)
     1-2.*G*(-3.*RO5/20.+3.*RO*RI4/4.-3.*RI5/5.)
      F(2,7)=B*(RO6/2.+RO2*RI4/2.-RI6)-2.*G*(-RO6/4.+3.*RO2*RI4/4.-RI6/
     12.)
      F(3,7)=B*(9.*RO7/14.+RO3*RI4/2.-8.*RI7/7.)
     1-2.*G*(-9.*RO7/28.+3.*RO3*RI4/4.-3.*RI7/7.)
      F(4,7)=2.*A*(RO5-RI5)/5.
      F(5,7)=A*(RO6-RI6)/3.+3.*G*ZL2*(RC4-RI4)/5.
      F(6,7)=2.*A*(RO7-RI7)/7.+24.*G*ZL2*(RO5-RI5)/25.
      F(7,7)=A*(RO8-RI8)/4.+6.*G*ZL2*(RC6-RI6)/5.
      F(7,1)=F(1,7)
      F(7,2)=F(2,7)
      F(7,3)=F(3,7)
      F(7,4)=F(4,7)
      F(7,5)=F(5,7)
      F(7,6)=F(6,7)
C
      F(2,1)=F(1,2)
      F(3,1)=F(1,3)
      F(3,2)=F(2,3)
      F(4,1)=F(1,4)
      F(4,2)=F(2,4)
      F(4,3)=F(3,4)
      F(5,1)=F(1,5)
      F(5,2)=F(2,5)
      F(5,3)=F(3,5)
      F(5,4)=F(4,5)
      F(6,1)=F(1,6)
      F(6,2)=F(2,6)
      F(6,3)=F(3,6)
      F(6,4)=F(4,6)
      F(6,5)=F(5,6)
C     WRITE(6,25)((F(K,J),J=1,7),K=1,7)
   25 FCRMAT(7D17.8)
   26 FORMAT(4E20.8)
      DO 80 I=1,7
      DC 80 J=1,7
      T(I,J)=F(I,J)
   80 CONTINUE
C
      CALL SYINV(F,7)
C
C     WRITE(6,25) ((F(K,J),J=1,7),K=1,7)
      DC 90 I=1,7
      DC 90 J=1,7
```

```
      FT(I,J)=0.
      CC 85 K=1,7
      FT(I,J)=FT(I,J)+T(I,K)*F(K,J)
   85 CCNTINUE
   SC CCNTINUE
      WRITE(6,25) ((FT(L,M),M=1,7),L=1,7)
      Y(1)=0.
      Y(2)=0.
      Y(3)=0.
      Y(4)=RHS4
      Y(5)=RHS5
      Y(6)=RHS6
      Y(7)=RHS7
C
      DC 100 I=1,7
      S(I)=0.
      DO 100 J=1,7
      S(I)=S(I)+F(I,J)*Y(J)
  10C CCNTINUE
      WRITE(6,25) (S(K),K=1,7)
C
C   STRESSES AT X=0, R=RC AND TAU AT X=0, R=RI
C
      ALPHA=-S(1)*RO-S(2)*RC2-S(3)*RO3
      WRITE(6,105) ALPHA
  105 FORMAT(5X,'ALPHA=',E20.8)
      WRITE(6,107)
  1C7 FORMAT(1H 2X,'STRAINS AND STRESSES')
      ER=+ZL*(S(1)+2.*S(2)*RO+3.*S(3)*RO2)
      ETHETA=ZL*(ALPHA/RC+S(1)+S(2)*RO+S(3)*RO2)
      EX=ZL*(S(4)+S(5)*RO+S(6)*RO2+S(7)*RO3)
      GAMRX=-(ALPHA+S(1)*RI+S(2)*RI2+S(3)*RI3)
      WRITE(6,26) ER,ETHETA,EX,GAMRX
C
      SIGR=A*ER+B*(EX+ETHETA)
      SIGO=A*ETHETA+B*(EX+ER)
      SIGX=A*EX+B*(ER+ETHETA)
      TAU=G*GAMRX
      WRITE(6,26) SIGR,SIGO,SIGX,TAU
C
      RE=1.49999
      RE2=RE**2
      ERE=ZL*( S(1)+2.*S(2)*RE+3.*S(3)*RE)
      ETHE=ZL*(ALPHA/RE+S(1)+S(2)*RE+S(3)*RE2)
      EXE=ZL*(S(4)+S(5)*RE+S(6)*RE2+S(7)*RE*RE2)
      SIGRE=A*ERE+B*(EXE+ETHE)
      WRITE(6,888) SIGRE
  888 FORMAT(5X,'RADIAL STRESS AT R=1.49999=',E20.8)
```

```
      RC=(RO+RI)/2.
      RC2=RC**2
      ERC=ZL*(S(1)+2.*S(2)*RC+3.*S(3)*RC2)/2.
      ETHC=ZL*(ALPHA/RC+S(1)+S(2)*RC+S(3)*RC2)/2.
      EXC=ZL*(S(4)+S(5)*RC+S(6)*RC2+S(7)*RC*RC2)/2.
      SIGRC=A*ERC+B*(ETHC+EXC)
      WRITE(6,889) SIGRC
  889 FCRMAT(5X,'RADIAL STRESS AT CENTER=',E20.8)
      ZR=1.499
      ZX=.001
      ZR2=ZR**2
      ERZ=(ZL-ZX)*(S(1)+2.*S(2)*ZR+3.*S(3)*ZR2)
      ETHZ=(ZL-ZX)*(ALPHA/ZR+S(1)+S(2)*ZR+S(3)*ZR2)
      EXZ=(ZL-ZX)*(S(4)+S(5)*ZR+S(6)*ZR2+S(7)*ZR*ZR2)
      SIGRZ=A*ERZ+B*(ETHZ+EXZ)
      WRITE(6,901) ZR,ZX,SIGRZ
  901 FCRMAT(5X,'AT R=',F10.5,'AND X=',F10.5,'THE RADIAL STRESS =',E20
     1.8)
C
      DC 150 K=1,NR
      SIGXX(K)=ZL*(A*(S(4)+S(5)*R(K))+B*(ALPHA/R(K)+2.*S(1)
     1+3.*S(2)*R(K)+4.*S(3)*(R(K)**2)))
      SIGXX(K)=SIGXX(K)+ZL*A*(S(6)*(R(K)**2)+S(7)*(R(K)**3))
  15C CCNTINUE
      WRITE(6,151) (SIGXX(K),K=1,NR)
      SIGXAV=0.
      DO 160 K=1,NR
      SIGXAV=SIGXAV+SIGXX(K)
  160 CCNTINUE
      SIGXAV=SIGXAV/NR
      WRITE(6,161) SIGXAV
  161 FORMAT(5X,'AVERAGE AXIAL STRESS=',E17.8)
  151 FORMAT(5X,6E17.8)
C
      FD=ZL*(A*S(4)*(RO2-RI2)/2.+B*(ALPHA*(RO-RI)+S(1)*(RO2-RI2)
     1+S(2)*(RO3-RI3)+S(3)*(RO4-RI4))+A*S(5)*(RC3-RI3)/3.)
      FC=FD+A*ZL*S(6)*(RO4-RI4)/4.+A*ZL*S(7)*(RC5-RI5)/5.
      FU=ZL*RHC*AMAX*(RC2-RI2)/2.
      PCTDIF=(1.+FD/FU)*100.
      WRITE(6,205) FD,FU,PCTDIF
  205 FORMAT( 5X,'FD=',E17.8,5X,'FU=',E17.8,5X,'PCTDIF=',E17.8)
C
      CC 300 I=1,11
      ZX=ZL*(I-1)/10
      ER=(ZL-ZX)*(S(1)+2.*S(2)*RO+3.*S(3)*RO2)
      ET=(ZL-ZX)*(ALPHA/RC+S(1)+S(2)*RO+S(3)*RO2)
      EX=(ZL-ZX)*(S(4)+S(5)*RO+S(6)*RC2+S(7)*RO3)
      SR=A*ER+B*(ET+EX)
```

```
C         WRITE(6,301) ZX,ER,ET,EX,SR
  301 FORMAT(5X,'AT X=',F10.5,5X,'ER=',E17.8,5X,'ETHETA=',E17.8,5X,'EX='
     1,E17.8,5X,'SR=',E17.8)
      U=(ZL-ZX)*(ALPHA+S(1)*RI+S(2)*RI2+S(3)*RI3)
      W=ZX*(ZL-ZX/2.)*(S(4)+S(5)*RI+S(6)*RI2+S(7)*RI3)
      WRITE(6,302) ZX,U,h,SR
  302 FCRMAT(5X,'AT X=',F10.5,5X,'U=',E17.8,5X,'h=',E17.8,5X,'SR=',E17
     1.8)
  30C CCNTINUE
  200 CCNTINUE
      STCP
C
      ENC
```